

Life Table

Life Table

- ▶ First described in 1958 by Cutler and Ederer, life-table analysis provides a straight forward, easy-to-perform method of analyzing survival.
- ▶ A life table presents the proportion surviving, The analysis accounts for subjects who die (fail) as well as subjects who are censored (withdrawn).
- ▶ The life-table method competes with the Kaplan-Meier product-limit method as a technique for survival analysis.
- ▶ The life-table method was developed first, but the Kaplan-Meier method has been shown to be superior and with the advent of computers is now the method of choice.
- ▶ However, for large samples, the life-table method is still popular in that it provides a simple summary of a large set of data.

Life Table

- ▶ In Life table assumes that withdrawals occur randomly throughout the interval and, if the sample size is large, will occur, on average, halfway between each interval.
- ▶ It also assumes that the probability of survival at one interval, though conditional on surviving previous intervals, is independent of the probability of survival at the prior interval(s).
- ▶ The first assumption is easily fulfilled when the intervals are short. As the length of the interval increases, however, this assumption becomes less likely.
- ▶ Life-table analysis can also be presented graphically as a survival curve that plots time versus cumulative survival.
- ▶ An investigator may be interested in studying patients who can enter a study at any point. The investigator may wish to orient the study around when a patient dies rather than when a predetermined interval is completed. The Kaplan-Meier product-limit method of survival analysis can be used in these cases.

Life Table

- ▶ A life table is constructed from a set of grouped or ungrouped failure data.
- ▶ The columns of the table are created using a set of formulas.
- ▶ The rows of the table represent various time intervals.

Each time interval is represented by $T_t \leq T < T_{t+1}$ or $[T_t, T_{t+1})$, where $t = 1, \dots, s$. The interval is from T_t up to but not including T_{t+1} . The intervals are assumed to be fixed. The intervals do not have to be of equal length, but it is often convenient to make them so.

The midpoint of the interval, T_{mt} , is defined as halfway through the interval.

The width of the interval is b_t where $b_t = T_{t+1} - T_t$. The width of the last interval, b_s , is theoretically infinite, so items requiring this value will be left blank.

Life Table

▶ **Number Lost to Follow-Up**

- ▶ The number lost to follow-up, l_t , is the number of individuals who were loss to observation during this interval, so their survival status is unknown.

▶ **Number Withdrawn Alive**

- ▶ The number withdrawn alive, $.w_t.$, is the number of individuals who had not died (failed) by the end of the study.

▶ **Number Dying**

- ▶ The number dying, $.d_t.$, is the number of individuals who die (fail) during the interval.

Life Table

► Number Entering the t^{th} Interval

The number entering the t^{th} interval, n'_t , is computed as follows. In the first interval, it is the total sample size. In the remaining intervals, it is computed using the formula

$$n'_t = n'_{t-1} - l_{t-1} - w_{t-1} - d_{t-1}$$

► Number Exposed to Risk

The number exposed to risk, n_t , is computed using the formula

$$n_t = n'_{t-1} - \frac{1}{2}(l_{t-1} + w_{t-1})$$

- This formula assumes that times to loss or withdrawal are distributed uniformly across the interval.

Life Table

► Conditional Proportion Dying

- The conditional proportion dying, q_t , is an estimate of the conditional probability of death in the interval given exposure to the risk of death in the interval. It is computed using the formula

- $q_t = \frac{d_t}{n_t}$

► Conditional Proportion Surviving

- The conditional proportion surviving, p_t , is an estimate of the conditional probability of surviving through the
- interval. It is computed using the formula

- $p_t = 1 - q_t$

Life Table

- ▶ **Cumulative Proportion Surviving**

- ▶ The cumulative proportion surviving, $S(T_t)$, is an estimate of cumulative survival rate at time T_t . It is computed using the formula

- ▶
$$S(T_t) = S(T_{t-1}) \times p_t$$

- ▶ Where; $S(T_0) = 1$

- ▶ The variance of this estimate is itself estimated using the formula (Green Wood)

- ▶
$$V[S(T)] = S(T_t)^2 \sum_{j=1}^{t-1} \frac{d_j}{(r_j - d_j)r_j}$$

- ▶ **Note:** is not used in generating the confidence intervals because it can produce intervals outside 0 and 1.

Life Table

(Kalbfleisch and Prentice 2002, 10, 15). Greenwood's formula for the asymptotic standard error of S_j is

$$s_j = S_j \sqrt{\sum_{k=1}^j \frac{d_k}{n_k(n_k - d_k)}}$$

(Greenwood 1926; Kalbfleisch and Prentice 2002, 17). s_j is reported as the standard deviation of survival but is not used in generating the confidence intervals because it can produce intervals outside 0 and 1. The “natural” units for the survivor function are $\log(-\log S_j)$, and the asymptotic standard error of that quantity is

$$\widehat{s}_j = \sqrt{\frac{\sum d_k / \{n_k(n_k - d_k)\}}{[\sum \log\{(n_k - d_k)/n_k\}]^2}}$$

(Kalbfleisch and Prentice 2002, 18). The corresponding confidence intervals are $S_j^{\exp(\pm z_{1-\alpha/2} \widehat{s}_j)}$.

The cumulative failure time is defined as $G_j = 1 - S_j$, and thus the variance is the same as for S_j and the confidence intervals are $1 - S_j^{\exp(\pm z_{1-\alpha/2} \widehat{s}_j)}$.

- Using the same example

1 5⁺ 6 6 8 8⁺ 9 11⁺

calculate Life Table for 4month interval.

$$n_t = n'_{t-1} - \frac{1}{2}(l_{t-1} + w_{t-1})$$

$$(T_t) = S(T_{t-1}) \times p_t \text{ , } p_t = 1 - q_t = \left(1 - \frac{d_t}{n_t - \frac{1}{2}(l_{t-1} + w_{t-1})}\right)$$

Interval	total	Deaths	Lost	Survival	error	[95% conf. int.]
0 4	8	1	0	0.875		
4 8	7	2	1	0.606		
8 12	4	2	2	0.202		

$$S(T_1) = S(T_0) \times P(t_1) = 1 \times \left(1 - \frac{1}{8 - \frac{1}{2}(0)}\right) = 0.875$$

$$S(T_2) = S(T_1) \times P(t_2) = 0.875 \times \left(1 - \frac{2}{7 - \frac{1}{2}(1)}\right) = 0.606$$

$$S(T_3) = S(T_2) \times P(t_3) = 0.606 \times \left(1 - \frac{2}{4 - \frac{1}{2}(2)}\right) = 0.202$$

Interval	total	Deaths	Lost	Survival	error	[95% conf. int.]
0 4	8	1	0	0.875	0.117	
4 8	7	2	1	0.606	0.178	
8 12	4	2	2	0.202	0.175	

$$\blacktriangleright V[S(T)] = S(T_t)^2 \sum_{j=1}^{t-1} \frac{d_j}{(r_j - d_j)r_j}$$

$$V[S(T_1)] = S(T_1)^2 \times \frac{d_1}{(r_1 - d_1)r_1} = 0.875^2 \left(\frac{1}{8 \times 7} \right) = 0.0137 \text{ error} = 0.117$$

$$V[S(T_2)] = S(T_2)^2 \left(\frac{d_2}{(r_2 - d_2)r_2} + \frac{d_1}{(r_1 - d_1)r_1} \right) = 0.606^2 \times \left(\frac{2}{6.5 \times 4.5} + \frac{1}{8 \times 7} \right) = 0.0317 \text{ error} = 0.178$$

$$V[S(T_3)] = S(T_3)^2 \left(\frac{d_3}{(r_3 - d_3)r_3} + \frac{d_2}{(r_2 - d_2)r_2} + \frac{d_1}{(r_1 - d_1)r_1} \right) = 0.202^2 \times \left(\frac{2}{3 \times 1} + \frac{2}{6.5 \times 4.5} + \frac{1}{8 \times 7} \right) = 0.0307$$

error=0.175

Interval	total	Deaths	Lost	Survival	error	[95% conf. int.]
0 4	8	1	0	0.875	0.117	(0.3870,0.9814)
4 8	7	2	1	0.606	0.178	(0.2062,0.8531)
8 12	4	2	2	0.202	0.175	(0.0097,0.5755)

$$\blacktriangleright \hat{S}_j = \sqrt{\frac{\sum d_k / \{n_k(n_k - d_k)\}}{\sum [\ln\{(n_k - d_k)/n_k\}]^2}} S_j^{\exp(\pm Z_{1-\alpha/2} \times \hat{S}_j)}$$

$$\hat{S}_1 = \sqrt{\frac{1/\{8 \times 7\}}{[\ln\{7/8\}]^2}} = 1.0007, (0.875^{\exp(1.96 \times 1.0007)}, 0.875^{\exp(-1.96 \times 1.0007)}) \quad (0.3870, 0.9814)$$

$$\hat{S}_2 = \sqrt{\frac{1/\{8 \times 7\} + 2/\{6.5 \times 4.5\}}{[\ln\{7/8\} + \ln\{4.5/6.5\}]^2}} = 0.5858 \quad (0.606^{\exp(1.96 \times 0.4327)}, 0.606^{\exp(-1.96 \times 0.5858)}) \quad (0.2062, 0.8531)$$

$$\hat{S}_3 = \sqrt{\frac{1/\{8 \times 7\} + 2/\{6.5 \times 4.5\} + 2/\{3 \times 1\}}{[\ln\{7/8\} + \ln\{4.5/6.5\} + \ln\{\frac{1}{3}\}]^2}} = 0.5424 \quad (0.202^{\exp(1.96 \times 0.3655)}, 0.202^{\exp(-1.96 \times 0.3655)}) \quad (0.0097, 0.5755)$$

Stata Out Put

► ltable time fail, survival intervals(4)

Interval		Beg. total	Deaths	Lost	Survival	Std. error	[95% conf. int.]	
0	4	8	1	0	0.8750	0.1169	0.3870	0.9814
4	8	7	2	1	0.6058	0.1779	0.2059	0.8530
8	12	4	2	2	0.2019	0.1752	0.0097	0.5754

Example of calculation Life Table

- ▶ McIlmurray and Turkie (2) describe a clinical trial of 49 patients for the treatment of Dukes' C colorectal cancer. The data for the two treatments, linoleic acid or control are given

Table 12.1 Survival in 49 patients with Dukes' C colorectal cancer randomly assigned to either γ linoleic acid or control treatment	
Treatment	Survival time (months)
γ linoleic acid (n=25)	1+, 5+, 6, 6, 9+, 10, 10, 10+, 12, 12, 12, 12, 12+, 13+, 15+, 16+, 20+, 24, 24+, 27+, 32, 34+, 36+, 36+, 44+
Control (n=24)	3+, 6, 6, 6, 6, 8, 8, 12, 12, 12+, 15+, 16+, 18+, 18+, 20, 22+, 24, 28+, 28+, 28+, 30, 30+, 33+, 42

- ▶ The + sign indicates censored data.
- ▶ We calculate Life Table for 12month interval.

Life Table with 12month Interval

$$V[S(T)] = S(T_t)^2 \sum_{j=1}^{t-1} \frac{d_j}{(r_j - d_j)r_j}$$

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Interval	total	Deaths	Lost	Survival	error	[95% conf. int.]
0	12	25	4	4	0.8261	0.0790
12	24	17	4	5	0.5982	0.1126
24	36	8	2	3	0.4141	0.1334
36	48	3	0	3	0.4141	0.1334

$$S(T_1) = S(T_0) \times P(t_1) = 1 \times \left(1 - \frac{4}{25 - \frac{1}{2}(4)}\right) = \frac{19}{23} = 0.8261$$

$$S(T_t) = S(T_{t-1}) \times p_t \quad \text{و} \quad p_t = 1 - q_t$$

$$S(T_2) = S(T_1) \times P(t_2) = 0.8261 \times \left(1 - \frac{4}{17 - \frac{1}{2}(5)}\right) = 0.5982$$

$$S(T_3) = 0.5982 \times \left(1 - \frac{2}{8 - \frac{1}{2}(3)}\right) = 0.4141$$

$$S(T_4) = 0.4141 \times \left(1 - \frac{0}{3 - \frac{1}{2}(3)}\right) = 0.4141$$

Life Table

$$V[S(T)] = S(T_t)^2 \sum_{j=1}^{t-1} \frac{d_j}{(r_j - d_j)r_j}$$

$$V[S(T_1)] = S(T_1)^2 \times \frac{d_1}{(r_1 - d_1)r_1} = 0.8261^2 \left(\frac{4}{19 \times 23} \right) = 0.00625 \quad \text{error} = \sqrt{0.00625} \\ = 0.0790$$

$$V[S(T_2)] = S(T_2)^2 \left(\frac{d_2}{(r_2 - d_2)r_2} + \frac{d_1}{(r_1 - d_1)r_1} \right) = 0.5982^2 \times \left(\frac{4}{10.5 \times 14.5} + \frac{4}{19 \times 23} \right) = 0.01268 \\ \text{error} = 0.1126$$

$$V[S(T_3)] = V[S(T_4)] = S(T_3)^2 \left(\frac{d_3}{(r_3 - d_3)r_3} + \frac{d_2}{(r_2 - d_2)r_2} + \frac{d_1}{(r_1 - d_1)r_1} \right) = 0.4141^2 \times \left(\frac{2}{4.5 \times 6.5} + \frac{4}{10.5 \times 14.5} + \frac{4}{19 \times 23} \right) \\ = 0.0178 \quad \text{error} = 0.1334$$

Interval	total	Deaths	Lost	Survival	error	[95% conf. int.]		
0 12	25	4	4	0.8261	0.0790	0.6006	0.9309	
12 24	17	4	5	0.5982	0.1126	0.3487	0.7783	
24 36	8	2	3	0.4141	0.1334	0.1646	0.6501	
36 48	3	0	3	0.4141	0.1334	0.1646	0.6501	

$$\hat{S}_j = \sqrt{\frac{\sum d_k / \{n_k(n_k - d_k)\}}{\sum [\ln\{(n_k - d_k)/n_k\}]^2}} S_j^{\exp(\pm Z_{1-\alpha/2} \times \hat{S}_j)}$$

$$\hat{S}_1 = \sqrt{\frac{4/\{23 \times 19\}}{[\ln\{19/23\}]^2}} = 0.5008, (0.8261^{\exp(1.96 \times 0.5008)}, 0.8261^{\exp(-1.96 \times 0.5008)}) \\ (0.6006, 0.9309)$$

$$\hat{S}_2 = \sqrt{\frac{4/\{23 \times 19\} + 4/\{10.5 \times 14.5\}}{[\ln\{19/23\} + \ln\{10.5/14.5\}]^2}} = 0.3663 (0.5982^{\exp(1.96 \times 0.3663)}, 0.5982^{\exp(-1.96 \times 0.3663)}) \\ (0.3487, 0.7783)$$

$$\hat{S}_3 = \sqrt{\frac{4/\{23 \times 19\} + 4/\{10.5 \times 14.5\} + \{2/4.5 \times 6.5\}}{[\ln\{19/23\} + \ln\{10.5/14.5\} + \ln\{4.5/6.5\}]^2}} = 0.3655 (0.4141^{\exp(1.96 \times 0.3655)}, \\ 0.5982^{\exp(-1.96 \times 0.3655)}) (0.1646, 0.6501)$$

Life Table with 8month Interval

Interval	total Deaths		Lost Survival		error	[95% conf. int.]			

0	8	25	2	2	0.9167	0.0564	0.7061	0.9785	
8	16	21	6	5	0.6194	0.1068	0.3791	0.7893	
16	24	10	0	2	0.6194	0.1068	0.3791	0.7893	
24	32	8	1	2	0.5309	0.1228	0.2736	0.7339	
32	40	5	1	3	0.3792	0.1553	0.1087	0.6547	
40	48	1	0	1	0.3792	0.1553	0.1087	0.6547	

$$S(T_t) = S(T_{t-1}) \times p_t$$

$$S(T_1) = S(T_0) \times p(t_1) = 1 \times \left(1 - \frac{2}{25 - 1/2(2)}\right) = 1 - \frac{2}{24} = 0.9167$$

$$S(T_2) = S(T_1) \times P(t_2) = 0.9167 \times \left(1 - \frac{6}{21 - 1/2(5)}\right) = 0.6194$$

$$S(T_3) = S(T_2) \times P(t_3) = 0.6194 \times \left(1 - \frac{0}{10 - 1/2(2)}\right) = 0.6194$$

$$S(T_4) = S(T_3) \times P(t_4) = 0.6194 \times \left(1 - \frac{1}{8 - 1/2(2)}\right) = 0.5309$$

$$S(T_5) = S(T_4) \times P(t_5) = 0.5309 \times \left(1 - \frac{1}{5 - 1/2(3)}\right) = 0.3792$$

$$S(T_6) = S(T_5) \times P(t_6) = 0.3792 \times \left(1 - \frac{0}{1 - 1/2(1)}\right) = 0.3792$$

$$V[S(T)] = S(T_t)^2 \sum_{j=1}^{t-1} \frac{d_j}{(r_j - d_j)r_j}$$

$$V[S(T_1)] = S(T_1)^2 \times \frac{d_1}{(r_1 - d_1)d_1} = 0.9167^2 \times \left(\frac{2}{22 \times 24}\right) = 0.003183 \quad \text{error} \\ = \sqrt{0.003183} = 0.0564$$

$$V[S(T_2)] = S(T_2)^2 \times \left(\frac{d_2}{(r_2 - d_2)d_2} + \frac{d_1}{(r_1 - d_1)d_1}\right) = 0.6194^2 \times \left(\frac{6}{125 \times 18.5} + \frac{2}{22 \times 24}\right) = 0.003183 \quad \text{error} \\ = \sqrt{0.003183} = 0.0564$$

$$\blacktriangleright V[S(T_4)] = S(T_4)^2 \left(\frac{d_4}{(r_4 - d_4)d_4} + \frac{d_3}{(r_3 - d_3)d_3} + \frac{d_2}{(r_2 - d_2)d_2} + \frac{d_1}{(r_1 - d_1)d_1}\right) = 0.5309^2 \left(\frac{1}{6 \times 7} + \frac{0}{125 \times 18.5} + \frac{2}{22 \times 24}\right) = 0.0151 \quad \text{error} = \sqrt{0.0151} = 0.1228$$

$$\blacktriangleright V[S(T_5)] = S(T_5)^2 \left(\frac{d_5}{(r_5 - d_5)d_5} + \frac{d_4}{(r_4 - d_4)d_4} + \frac{d_3}{(r_3 - d_3)d_3} + \frac{d_2}{(r_2 - d_2)d_2} + \frac{d_1}{(r_1 - d_1)d_1}\right) = 0.3792^2 \left(\frac{1}{2.5 \times 3.5} + \frac{1}{6 \times 7} + \frac{0}{125 \times 18.5} + \frac{2}{22 \times 24}\right) = 0.0241 \quad \text{error} = \sqrt{0.0241} = 0.1553$$

Interval	total Deaths	Lost Survival	error	[95% conf. int.]
0 8	25	2 2	0.9167 0.0564	0.7061 0.9785
8 16	21	6 5	0.6194 0.1068	0.3791 0.7893
16 24	10	0 2	0.6194 0.1068	0.3791 0.7893
24 32	8	1 2	0.5309 0.1228	0.2736 0.7339
32 40	5	1 3	0.3792 0.1553	0.1087 0.6547
40 48	1	0 1	0.3792 0.1553	0.1087 0.6547

$$\hat{S}_j = \sqrt{\frac{\sum d_k / \{n_k(n_k - d_k)\}}{\sum [\ln\{(n_k - d_k)/n_k\}]^2}} S_j^{\exp(\pm Z_{1-\alpha/2} \times \hat{S}_j)}$$

$$\hat{S}_1 = \sqrt{\frac{2/\{24 \times 22\}}{[\ln\{22/24\}]^2}} = 0.7073, (0.9167^{\exp(1.96 \times 0.7073)}, 0.8261^{\exp(-1.96 \times 0.7073)}) (0.7061, 0.9785)$$

$$\hat{S}_2 = \sqrt{\frac{4/\{23 \times 19\} + 6/\{12.5 \times 18.5\}}{[\ln\{19/23\} + \ln\{12.5/18.5\}]^2}} = 0.3699 (0.6194^{\exp(1.96 \times 0.3699)}, 0.6194^{\exp(-1.96 \times 0.3699)}) (0.3791, 0.7893)$$

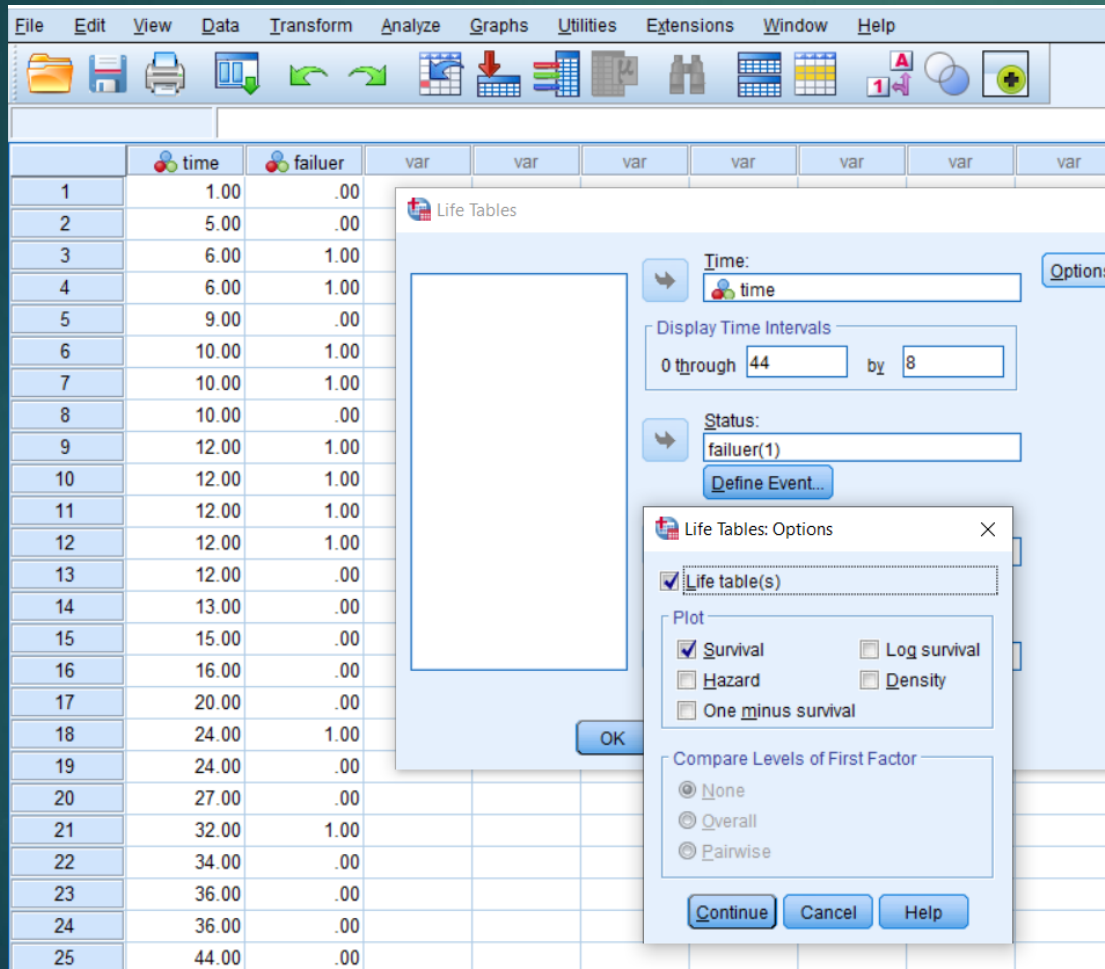
$$\hat{S}_4 = \sqrt{\frac{4/\{23 \times 19\} + 6/\{12.5 \times 18.5\} + 1/\{6 \times 7\}}{[\ln\{19/23\} + \ln\{12.5/18.5\} + \ln\{6/7\}]^2}} = 0.3654 (0.5309^{\exp(1.96 \times 0.3654)}, 0.2736^{\exp(-1.96 \times 0.3654)}) (0.2736, 0.7339)$$

$$\hat{S}_5 = \sqrt{\frac{4/\{23 \times 19\} + 6/\{12.5 \times 18.5\} + 1/\{6 \times 7\} + 1/\{2.5 \times 3.5\}}{[\ln\{19/23\} + \ln\{12.5/18.5\} + \ln\{6/7\} + \ln\{2.5/3.5\}]^2}} = 0.3654 (0.5309^{\exp(1.96 \times 0.3654)}, 0.2736^{\exp(-1.96 \times 0.3654)})$$

SPSS

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► Analysis/Survival Analysis/ Life table



The image shows the SPSS Life Tables dialog box and its options. The background is a data editor with columns 'time' and 'failuer' (likely 'failure'). The 'Time' variable is selected in the 'Time' field, and 'failuer(1)' is selected in the 'Status' field. The 'Display Time Intervals' section shows '0 through 44 by 8'. The 'Life Tables: Options' sub-dialog is open, showing 'Life table(s)' checked, 'Survival' selected under 'Plot', and 'None' selected under 'Compare Levels of First Factor'.

	time	failuer	var	var	var	var	var	var
1	1.00	.00						
2	5.00	.00						
3	6.00	1.00						
4	6.00	1.00						
5	9.00	.00						
6	10.00	1.00						
7	10.00	1.00						
8	10.00	.00						
9	12.00	1.00						
10	12.00	1.00						
11	12.00	1.00						
12	12.00	1.00						
13	12.00	.00						
14	13.00	.00						
15	15.00	.00						
16	16.00	.00						
17	20.00	.00						
18	24.00	1.00						
19	24.00	.00						
20	27.00	.00						
21	32.00	1.00						
22	34.00	.00						
23	36.00	.00						
24	36.00	.00						
25	44.00	.00						

SPSS Out Put

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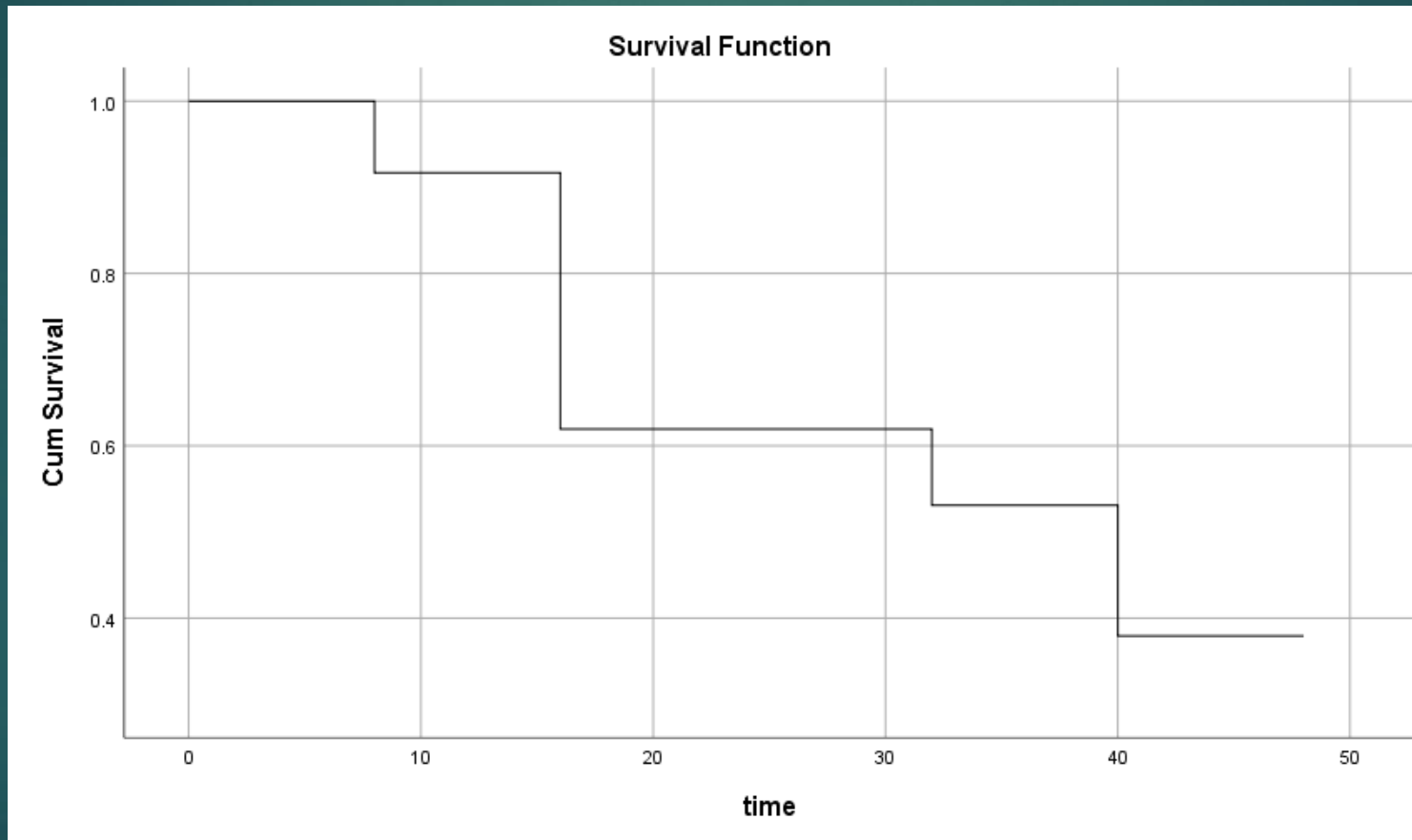
Life Table^a

Interval Start Time	Number Entering Interval	Number Withdrawing during Interval	Number Exposed to Risk	Number of Terminal Events	Proportion Terminating	Proportion Surviving	Cumulative Proportion Surviving at End of Interval	Std. Error of Cumulative Proportion Surviving at End of Interval
0	25	2	24.000	2	.08	.92	.9167	.0564
8	21	5	18.500	6	.32	.68	.6194	.1068
16	10	2	9.000	0	.00	1.00	.6194	.1068
24	8	2	7.000	1	.14	.86	.5309	.1228
32	5	3	3.500	1	.29	.71	.3792	.1553
40	1	1	.500	0	.00	1.00	.3792	.1553

a. The median survival time is 33.6291

SPSS Out Put, Life Table Graph

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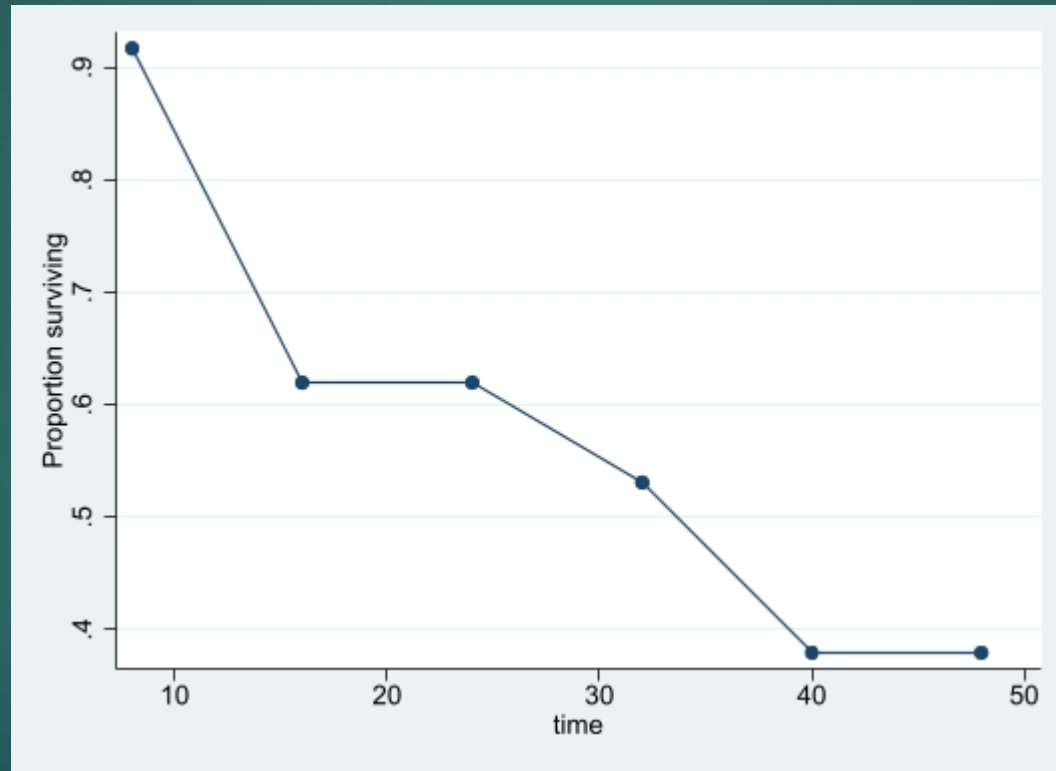


STATA: Itable time failuer, graph survival intervals(8)

Interval		Beg. total	Deaths	Lost	Survival	Std. error	[95% conf. int.]	
0	8	25	2	2	0.9167	0.0564	0.7061	0.9785
8	16	21	6	5	0.6194	0.1068	0.3791	0.7893
16	24	10	0	2	0.6194	0.1068	0.3791	0.7893
24	32	8	1	2	0.5309	0.1228	0.2736	0.7339
32	40	5	1	3	0.3792	0.1553	0.1087	0.6547
40	48	1	0	1	0.3792	0.1553	0.1087	0.6547

STATA Out Put, Life Table Graph

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Life Table OR Kaplan Meier

- ▶ The Life Table method is very useful for a large sample, but the estimated results will depend on the chosen interval length. The larger the interval, the poorer the estimations.
- ▶ Life-table analysis provides easy-to-understand and easily performed analysis of data obtained at pre-determined intervals.
- ▶ The Kaplan-Meier product-limit method uses exact survival times rather than intervals to analyze survival.
- ▶ You should apply Kaplan-Meier method if the sample is not very large.

Question

- ▶ Difference between K-P and Life table?